Quasi-Monte Carlo Simulation of the Light Environment of Virtual Plants

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Introduction

The distribution of light in the canopy is a major factor regulating the growth and development of a plant. Consequently, simulation of light environment is an important component of functional-structural plant modelling. The main variables of interest are the quantity of photosynthetically active radiation (PAR) reaching different elements of the plant canopy, and the quality (spectral composition) of light reaching those elements, which is a signal for photomorphogenesis.

Light environment models estimate the irradiance reaching a plant from direct light sources (e.g., sun and sky), and often include indirect sources (e.g., light reflected from/transmitted through plant organs). These estimates rely on the description of the plant canopy, which may be approximated as a turbid-medium or specified explicitly as a geometric structure (virtual plant). Models of light environment at the plant structure level are usually based either on the Monte Carlo path tracing method or the radiosity method (Chelle and Andrieu, 2007). In the past, attention was given to improve the efficiency of the radiosity method (Chelle and Andrieu, 1998; Soler *et al.*, 2003), with Monte Carlo path tracing used as a benchmark for comparison. Here, we focus on improving the accuracy and efficiency of path tracing.

Monte Carlo path tracing is derived from standard ray tracing. Both methods approximate the solution to the rendering equation, which describes the transfer of light energy between one point on a surface to another (Kajiya, 1986). Path tracing differs from ray tracing by following many single light paths from a surface point instead of recursively following a single reflecting and refracting ray. Thus, path tracing captures some optical phenomena, such as diffuse light reflection and transmission, which ray-tracing does not.

A ray is traced through the plant canopy until all of its radiant energy is absorbed by the plant's organs. When the ray intersects an organ, a local light model is applied to calculate how much light is reflected, transmitted, or absorbed by that organ. In Monte Carlo path tracing (Kajiya, 1986), a reflected or transmitted ray is then generated stochastically. The direction and energy of these rays depends on the bidirectional reflectance distribution function (BRDF) or bidirectional transmittance distribution function (BTDF) of the organ's surface.

From a computational perspective, a light ray may originate from a light source and be traced towards the plant canopy, or from a plant organ and be traced backward towards a light source. When the canopy is dense, the former method is advantageous, because many rays originating at plant organs would never reach a light source. In contrast, when organs are small relative to the whole plant and are highly dispersed, the latter method is advantageous, because rays traced from a light source would often miss organs.

A light environment model based on Monte Carlo path tracing was developed by Měch (1997), and can be conveniently interfaced with virtual plants expressed using the open L-system formalism (Měch and Prusinkiewicz, 1996). Měch applied two variance-reduction techniques to calculate light distribution efficiently. First, rays are generated not with a uniform distribution, but preferentially in the direction from which (or towards which) most energy goes (importance sampling). Second, individual rays may carry information regarding several wavelengths simultaneously. This technique reduces variance of the ratios of energy associated with different light wavelengths, which is important in estimating the spectrum of light reaching plant organs (Měch, 1997; Gautier *et al.*, 2000).

To further improve the efficiency of the light distribution calculations provided by Měch's model, we have implemented an alternative method for generating reflected or transmitted rays, called the quasi-Monte Carlo method (surveyed in the context of computer graphics by Owen, 2003). While Monte Carlo (MC) path tracing relies on random sampling of the space of reflected or transmitted rays, and results in a set of independent paths, quasi-Monte Carlo (QMC) is based on a highly regular sampling that produces a set of correlated paths. As shown by Keller *et al.* (1996) and Veach (1997), this reduces the number of rays required for path-tracing virtual 3D scenes within given error bounds. Several algorithms for generating sets or sequences of regularly spaced sampling points have been proposed for use in QMC computations; the most commonly used in practice are by Korobov (1959), and Sobol' (1967), Halton (1960) and Faure (1982). We chose Korobov's algorithm because it can generate sampling points dynamically, as the tracing proceeds, without knowing in advance how many ray-surface intersections will occur in each path, and thus how many sampling points will be needed to trace it.

In principle, QMC methods are deterministic, but they can be randomized in a way that preserves highly regular distribution of the sampling points. This can be used to estimate error/variance of the computation (QMC paths are not independent, so simple error estimation as in MC is not possible otherwise). *Randomized quasi-Monte Carlo* methods can thus be seen as a general variance reduction technique that can be coupled with more problem-specific methods, such as importance sampling, to improve upon MC. These methods are used in our simulation program for approximating the error in computation.

The QuasiMC program for simulating light distribution in a canopy

The L-system-based plant model and the light environment model are executed as two separate processes that communicate using the open L-system formalism (Měch and Prusinkiewicz, 1996). The plant simulator, *cpfg/lpfg*, sends information about the location, size and orientation of the virtual plant's organs to the light environment, and the light environment simulator, *QuasiMC*, returns the lighting distribution among those organs. Figure 1 shows the two programs, *lpfg* and *QuasiMC*, from the user's perspective.

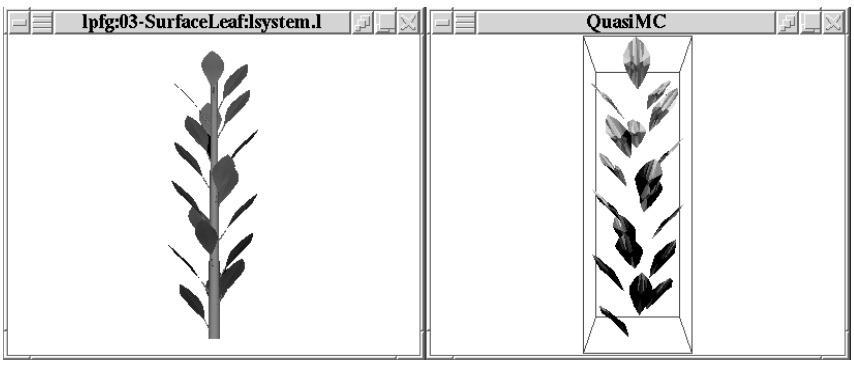

Figure 1: The plant-light simulation software from the user's perspective: on the left, the plant simulator *lpfg*, and on the right, the light environment simulator *QuasiMC*. The *QuasiMC* program shows the plant organs which are included in the light distribution simulation.

The *QuasiMC* program supports two types of light sources: directional light sources, with all rays having the same initial direction, and a hemispherical approximation of the sky based on the CIE standard clear sky model and overcast sky model (CIE 110, 1994). The user can specify parameters of either light source type, for example the radiant energy of each directional light source or the time of day for the sky model. The user can also specify which tracing method (from the light sources to the canopy or from the canopy to the sources) should be employed in the computations.

A plant organ is represented as a triangle, a parallelogram, or a Bezier surface. Each organ is associated with a set of user-specifiable parameters that characterize its optical properties according to the *Blinn-Phong* local illumination model (Měch, 1997, page 283). This model is not based on measured phytoelement data from a device such as a spectrogoniometer, but Chelle has shown the suitability of a similar model for simulating light absorption in plants (Chelle, 2006).

Similarly to Měch's *MonteCarlo* program, *QuasiMC* uses uniform spatial subdivision to speed-up computation. The benefit comes from the reduced time needed to find intersections between a light path and a plant organ. Where a basic method would test each organ for a possible intersection with a ray, the spatial subdivision method only tests those organs that are close to the ray.

Results and Discussion

We compared QMC and MC methods by performing numerical experiments on a virtual canopy of randomly distributed leaves. These experiments were similar to those by Chelle and Andrieu (1998, page 81). The canopy model consisted of 500 triangles uniformly distributed and oriented within a cube (Figure 2). Each triangle was set to behave as a Lambertian surface with 10% of the radiant energy reflecting from the surface and 10% transmitting through it. The scene was illuminated from above, using a directional light source. Each experiment involved the same number of light paths, 65,536. To estimate error in the results, the experiments were repeated 10 times for both the MC and QMC methods using the *QuasiMC* program (it is possible to use Monte Carlo sampling with this program). The results are shown in Figure 3.

The mean values of the irradiance reaching each leaf over the 10 simulations for the MC and QMC methods are similar (compare Fig. 2a with Fig. 2b), but the QMC method produced significantly smaller variance (compare Fig. 2c with Fig. 2d). Fitted curves are shown in all four graphs to highlight the overall trends. The mean variance over all 500 leaves in the MC sample is 75.46×10^{-5} with standard deviation 99.79×10^{-5}, while in the QMC sample it is 9.37×10^{-5} with standard deviation 10.78×10^{-5}. The computation time was about 4 seconds in both cases on a PC 3.0GHz computer, but even when the number of leaves was increased to 10,000, the computation time was only about a minute. The ultimate benefit of using QMC method is that fewer light paths must be computed to achieve the same level of accuracy as the Monte Carlo method. For example, in our experiment, four times fewer light paths needed to be traced by the QMC method compared to the Monte Carlo method (16,384 instead of 65,536) to calculate light distribution with approximately the same variance (for the QMC method with 16,384 rays, the mean variance was 67.39×10^{-5} with standard deviation 72.33×10^{-5}).

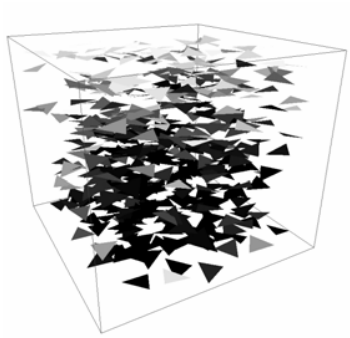

Figure 2: 3D canopy mock-up of 500 triangles uniformly distributed within a cube. Each triangle is shaded according to the amount of light reaching it, with lighter shades representing high irradiance and darker shades representing low irradiance.

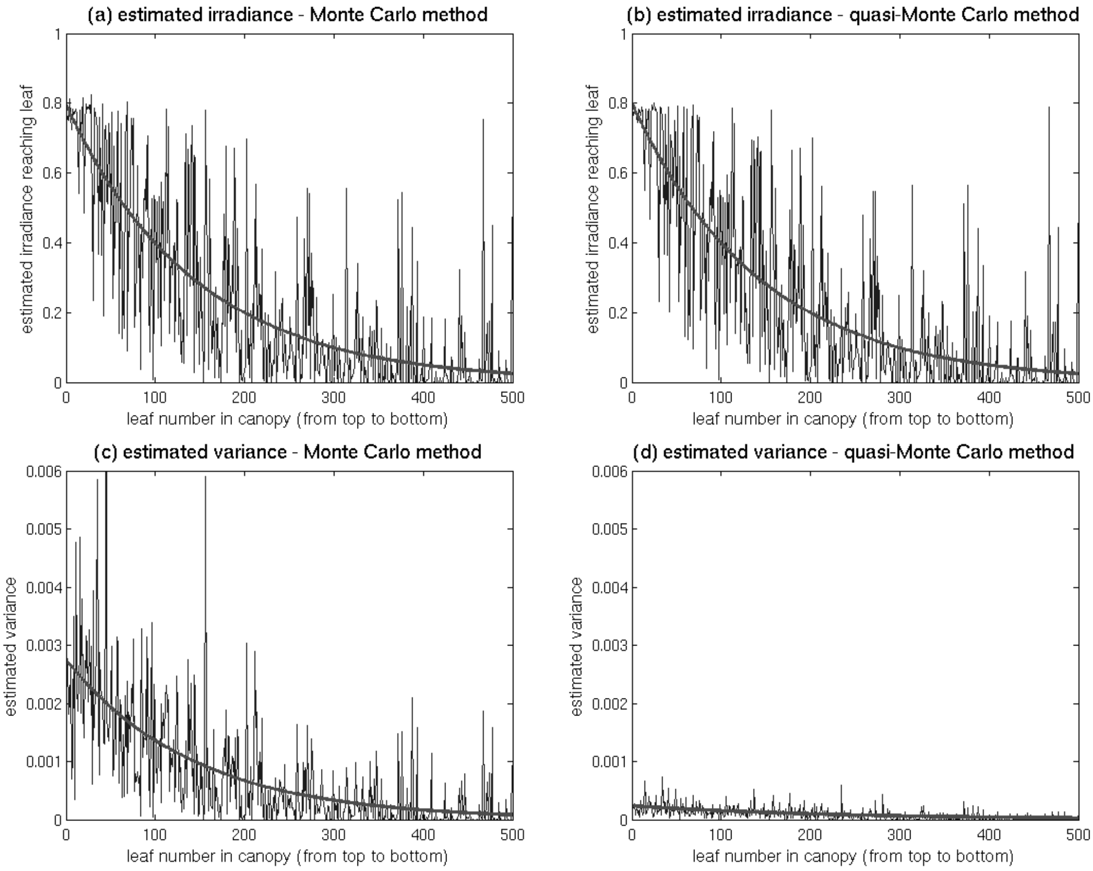

Figure 3: Comparison of estimated irradiances, reaching individual uniformly distributed leaves in a plant canopy model. The fitted curves are described by equations of the form $y = Ae^{-\lambda x}$. Parameters A and λ were estimated by minimizing the sum-of-squares error for $\sum_i Ae^{-\lambda x_i} - y_i$, where i is the leaf number. (a) Monte Carlo estimate of irradiance, (b) (randomized) quasi-Monte Carlo estimate of irradiance, (c) estimated variance in Monte Carlo calculation, and (d) estimated variance in (randomized) quasi-Monte Carlo calculation.

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