THE VIRTUAL LABORATORY

THE ALGORITHMIC BEAUTY OF PLANTS

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Aristid Lindenmayer 1925–1989

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The Algorithmic Beauty of Plants

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With 150 Illustrations, 48 in Color

This edition of *The Alogirthmic Beauty of Plants* is the electronic version of the book that was published by Springer-Verlag, New York, in 1990 and reprinted in 1996. The electronic version has been produced using the original LaTeX files and digital illustrations.

Front cover design: The roses in the foreground (*Roses* by D. R. Fowler, J. Hanan and P. Prusinkiewicz [1990]) were modeled using L-systems. Distributed ray-tracing with one extended light source was used to simulate depth of field. The roses were placed on a background image (photgraphy by G. Rossbach), which was scanned digitally and post-processed.

Preface

Mathematics and beauty

The beauty of plants has attracted the attention of mathematicians for centuries. Conspicuous geometric features such as the bilateral symmetry of leaves, the rotational symmetry of flowers, and the helical arrangements of scales in pine cones have been studied most extensively. This focus is reflected in a quotation from Weyl [159, page 3], "Beauty is bound up with symmetry."

This book explores two other factors that organize plant structures and therefore contribute to their beauty. The first is the elegance and relative simplicity of *developmental algorithms*, that is, the rules which describe plant development in time. The second is *self-similarity*, characterized by Mandelbrot [95, page 34] as follows:

> When each piece of a shape is geometrically similar to the whole, both the shape and the cascade that generate it are called self-similar.

This corresponds with the biological phenomenon described by Herman, Lindenmayer and Rozenberg [61]:

> In many growth processes of living organisms, especially of plants, regularly repeated appearances of certain multicellular structures are readily noticeable.... In the case of a compound leaf, for instance, some of the lobes (or leaflets), which are parts of a leaf at an advanced stage, have the same shape as the whole leaf has at an earlier stage.

Growth and form

Thus, self-similarity in plants is a result of developmental processes. By emphasizing the relationship between growth and form, this book follows a long tradition in biology. D'Arcy Thompson [143] traces its origins to the late seventeenth century, and comments:

> Organic form itself is found, mathematically speaking, to be a function of time.... We might call the form of an organism an *event in space-time*, and not merely a *configuration in space*.

This concept is echoed by Hallé, Oldeman and Tomlinson [58]:

> The idea of the form implicitly contains also the history of such a form.

Modeling of plants

The developmental processes are captured using the formalism of *L-systems*. They were introduced in 1968 by Lindenmayer [82] as a theoretical framework for studying the development of simple multicellular organisms, and subsequently applied to investigate higher plants and plant organs. After the incorporation of geometric features, plant models expressed using L-systems became detailed enough to allow the use of computer graphics for realistic visualization of plant structures and developmental processes.

The emphasis on graphics has several motivations. A visual comparison of models with real structures is an important component of model validation. The display of parameters and processes not observable directly in living organisms may assist in the analysis of their physiology, and thus present a valuable tool for educational purposes. From an aesthetic perspective, plants present a wealth of magnificent objects for image synthesis. The quest for photorealism challenges modeling and rendering algorithms, while a departure from realism may offer a fresh view of known structures.

The application of computer graphics to biological structures is only one of many factors that contribute to the interdisciplinary character of this book. For example, the notion of L-systems is a part of formal language theory, rooted in the theory of algorithms. The application of L-systems to plant description has been studied by biologists, and involves various methods of general mathematics. Self-similarity relates plant structures to the geometry of fractals. Computer-aided visualization of these structures, and the processes that create them, joins science with art.

About the book

The study of an area that combines so many disciplines is very stimulating. Some results may be of special interest to students of biology or computer graphics, but a much wider circle of readers, generally interested in science, may find mathematical plant models inspiring, and the open problems worth further thought. Consequently, all basic concepts are presented in a self-contained manner, assuming only general knowledge of mathematics at the junior college level.

This book focuses on original research results obtained by the authors in the scope of the cooperation between the Theoretical Biology Group, directed by Aristid Lindenmayer at the University of Utrecht, and the Computer Graphics Group, working under the supervision of Przemyslaw Prusinkiewicz at the University of Regina. Technically, the book evolved from the SIGGRAPH '88 and '89 course notes *Lindenmayer systems, fractals, and plants*, published by Springer-Verlag in the series *Lecture Notes in Biomathematics* [112]. The present volume has been extended with edited versions of recent journal and conference papers (see Sources), as well as previously unpublished results.

Aristid Lindenmayer is the author of the notion of L-systems which forms the main thread of the book. He also played an essential role in the reported research by suggesting topics for study, guiding the construction of specific plant models, monitoring their correctness and participating in many discussions of biological and mathematical problems. Seriously ill, Professor Lindenmayer co-authored and edited several chapters, but was not able to participate in the completion of this work. If any inaccuracies or mistakes remain, he could not prevent them. Still, in spite of unavoidable shortcomings, we hope that this book will convey his and our excitement of applying mathematics to explore the beauty of plants.

Acknowledgements

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Sources

In the preparation of this book, edited parts of the following publications were used:

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